

THE PROBABILISTIC STABILITY FOR A FUNCTIONAL NONLINEAR EQUATION IN A SINGLE VARIABLE

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Abstract. We use the fixed point method to prove the probabilistic Hyers–Ulam and generalized Hyers–Ulam–Rassias stability for the nonlinear equation $f(x) = \Phi(x, f(\eta(x)))$ where the unknown is a mapping f from a nonempty set S to a probabilistic metric space (X, F, T_M) and $\Phi : S \times X \rightarrow X$, $\eta : S \rightarrow X$ are two given functions.

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