

SOME INEQUALITIES RELATING TO UPPER AND LOWER BOUNDS FOR THE RIEMANN—STIELTJES INTEGRAL

S. S. DRAGOMIR AND C. E. M. PEARCE

Abstract. Some new inequalities are obtained relating to the generalized trapezoid and midpoint rules for the Riemann–Stieltjes integral with a convex integrand and monotone nondecreasing integrator. Results are deduced for the special case of weighted Riemann integrals.

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REFERENCES

- [1] G.A. ANASTASSIOU, *Grüss type inequalities for the Stieltjes integral*, *Nonlin. Funct. Anal. Appl.*, **12** (2007), 583–593.
- [2] G.A. ANASTASSIOU, *Chebyshev–Grüss type and comparison of integral means inequalities for the Stieltjes integral*, *Panam. Math. J.*, **17** (2007), 91–109.
- [3] N.S. BARNETT, W.-S. CHEUNG, S.S. DRAGOMIR AND A. SOFO, *Ostrowski and trapezoid type inequalities for the Stieltjes integral with Lipschitzian integrands or integrators*, Preprint, RGMIA Res. Rep. Coll. (2006) Article 9. <http://rgmia.vu.edu.au/v9n4.html>
- [4] P. CERONE, W.-S. CHEUNG AND S.S. DRAGOMIR, *On Ostrowski type inequalities for Stieltjes integrals with absolutely continuous integrands and integrators of bounded variation*, *Comput. Math. Appl.*, **54** (2007), 183–191.
- [5] P. CERONE AND S.S. DRAGOMIR, *New bounds for the three-point rule involving the Riemann–Stieltjes integral*, *Advances in Statistics, Combinatorics and Related Areas*, 53–62, World Sci. Publ., River Edge, NJ, 2002.
- [6] P. CERONE AND S.S. DRAGOMIR, *Approximation of the Stieltjes integral and applications in numerical integration*, *Appl. Math.*, **51** (2006), 37–47.
- [7] P. CERONE, S.S. DRAGOMIR AND C.E.M. PEARCE, *A generalized trapezoid inequality for functions of bounded variation*, *Turkish J. Math.*, **24** (2000), 147–163.
- [8] W.-S. CHEUNG AND S.S. DRAGOMIR, *Two Ostrowski type inequalities for the Stieltjes integral of monotonic functions*, *Bull. Austral. Math. Soc.*, **75** (2007), 299–311.
- [9] R. DARST AND H. POLLARD, *An inequality for the Riemann–Stieltjes integral*, *Proc. Amer. Math. Soc.*, **25** (1970), 912–913.
- [10] S.S. DRAGOMIR, *On the Ostrowski’s inequality for Riemann–Stieltjes integral*, *Korean J. Appl. Math.*, **7** (2000), 477–485.
- [11] S.S. DRAGOMIR, *An Ostrowski like inequality for convex functions and applications*, *Revista Math. Complutense*, **16** (2003), 373–382.
- [12] S.S. DRAGOMIR, *Inequalities of Grüss type for the Stieltjes integral and applications*, *Kragujevac J. Math.*, **26** (2004), 89–122.
- [13] S.S. DRAGOMIR, *On the Ostrowski inequality for Riemann–Stieltjes integral $\int_a^b f(t) du(t)$ where f is of Hölder type and u is of bounded variation and applications*, *J. KSIAM*, **5** (2001), 35–45.
- [14] S.S. DRAGOMIR, *Inequalities for Stieltjes integrals with convex integrators and applications*, *Appl. Math. Lett.*, **20** (2007), 123–130.
- [15] S.S. DRAGOMIR, C. BUSE, M.V. BOLDEA AND L. BRAESCU, *A generalization of the trapezoidal rule for the Riemann–Stieltjes integral and applications*, *Nonlinear Anal. Forum*, **6** (2001), 337–351.

- [16] S.S. DRAGOMIR AND I.A. FEDOTOV, *An inequality of Grüss' type for Riemann–Stieltjes integral and applications for special means*, Tamkang J. Math., **29** (1998), 287–292.
- [17] S.S. DRAGOMIR AND I. FEDOTOV, *A Grüss type inequality for mappings of bounded variation and applications to numerical analysis*, Nonlinear Funct. Anal. Appl., **6** (2001), 425–438.
- [18] ZHENG LIU, *Refinement of an inequality of Grüss type for Riemann–Stieltjes integral*, Soochow J. Math., **30** (2004), 483–489.
- [19] P.R. MERCER, *Hadamard's inequality and trapezoid rules for the Riemann–Stieltjes integral*, J. Math. Anal. Applic., **344** (2008), 921–926.
- [20] R.T. RAKHMAIL, *The inverse Chebyshev inequality for Stieltjes integrals*, Math. Anal. (Russian), 9–15 Leningrad Gos. Ped. Inst. (1990).