

MEAN THEORETIC APPROACH TO A FURTHER EXTENSION OF GRAND FURUTA INEQUALITY

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Abstract. Very recently, Furuta has shown a further extension of grand Furuta inequality. In this paper, we obtain a more precise and clear expression of Furuta's extension by considering a mean theoretic proof of grand Furuta inequality. Moreover, we get a variant of Furuta's extension by scrutinizing the former argument.

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