

INTEGRAL, DISCRETE AND FUNCTIONAL VARIANTS OF JENSEN'S INEQUALITY

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Abstract. We deal with the convex functions on the bounded closed convex sets with common barycenter. More precisely, the integral arithmetic means of convex function f are compared on these two sets A and B with $A \subset B$. The paper shows that series of inequalities

$$\frac{1}{\mu(A)} \int_A f(x) d\mu(x) \leq \frac{1}{\mu(B)} \int_B f(x) d\mu(x) \leq \frac{1}{\mu(B \setminus A)} \int_{B \setminus A} f(x) d\mu(x)$$

hold for convex functions of one variable, but it doesn't generally hold for convex functions of several variables. The article also gives discrete and functional variants of the mentioned inequality, and their applications to quasi-arithmetic and power means.

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