

ON A BETA FUNCTION INEQUALITY

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Abstract. In this paper we present a beta function inequality on $(0, 1] \times (0, 1]$, which improves an inequality of H. Alzer. Moreover several new inequalities for the gamma and psi functions on $(0, 1]$ are provided. Some elementary inequalities of two real variables are proved.

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