

ON IMPROVED ARITHMETIC–GEOMETRIC MEAN AND HEINZ INEQUALITIES FOR MATRICES

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Abstract. In this paper, we first generalize an inequality and improve another one for unitarily invariant norms, which are established by Kittaneh and Manasrah in [Improved Young and Heinz inequalities for matrices. *J. Math. Anal. Appl.* 361(2010)262–269]. Then we present a new inequality for unitarily invariant norms, which is equivalent to an inequality presented by Kittaneh and Manasrah in the case of the Hilbert-Schmidt norm.

Mathematics subject classification (2010): 15A18, 15A42, 15A60.

Keywords and phrases: unitarily invariant norms, arithmetic-geometric mean inequality, Heinz inequality, positive semidefinite matrix.

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