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MATRIX INEQUALITIES INCLUDING FURUTA INEQUALITY VIA RIEMANNIAN MEAN OF n–MATRICES

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Abstract. Very recently, Yamazaki has obtained an excellent generalization of Ando-Hiai inequality and a characterization of chaotic order (so called Furuta inequality for chaotic order) via weighted Riemannian mean, a kind of geometric mean, of n positive definite matrices.

In this paper, by discussing extensions of Yamazaki's results, we shall obtain a generalization of Furuta inequality via weighted Riemannian mean of n-matrices.

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