

SHARP BOUNDS FOR TOADER MEAN IN TERMS OF CONTRAHARMONIC MEAN WITH APPLICATIONS

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Abstract. We find the greatest value λ and the least value μ in $(1/2, 1)$ such that the double inequality $C(\lambda a + (1 - \lambda)b, \lambda b + (1 - \lambda)a) < T(a, b) < C(\mu a + (1 - \mu)b, \mu b + (1 - \mu)a)$ holds for all $a, b > 0$ with $a \neq b$, and give new bounds for the perimeter of an ellipse. Here,

$T(a, b) = \frac{2}{\pi} \int_0^{\pi/2} \sqrt{a^2 \cos^2 \theta + b^2 \sin^2 \theta} d\theta$, and $C(a, b) = (a^2 + b^2)/(a + b)$ denote the Toader, and contraharmonic means of two positive numbers a and b , respectively.

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