

## ANOTHER PROOF OF SPIRA'S INEQUALITY AND ITS APPLICATION TO THE RIEMANN HYPOTHESIS

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*Abstract.* By using new inequalities involving powers of rational functions, we give another proof of an important Spira's relation for the Riemann zeta-function  $|\zeta(1-s)| \leq |\zeta(s)|$  in the strip  $0 < \Re s < 1/2$ ,  $|\Im s| \geq 12$ . Moreover, we establish a sufficient condition of the validity of the Riemann hypothesis in terms of the derivative of  $|\zeta(s)|^2$  with respect to  $\Re s$  and conjecture its necessity.

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