

NEW REFINEMENTS OF TWO INEQUALITIES FOR MEANS

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Abstract. In paper [2] H. Alzer proved that the logarithmic mean of two distinct positive real numbers lies between the geometric mean and the arithmetic mean of the geometric and identric means of these numbers. Refinements of these inequalities were provided in [11]. In this note we offer refinements of a new type.

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