

## A HILBERT INTEGRAL INEQUALITY WITH HURWITZ ZETA FUNCTION

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*Abstract.* It is shown that an extension of the Hilbert integral inequality can be established by introducing two parameters and a proper logarithm function. The constant factor expressed by product of the gamma function and the Hurwitz Zeta function is proved to be the best possible. And base on it, some interesting special results are enumerated. As applications, some equivalent forms are given.

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