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A ONE–PARAMETER FAMILY OF BIVARIATE MEANS

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Abstract. A one-parameter family of bivariate means is introduced. Members of the new family of means are derived from a bivariate symmetric mean. It is shown that new means are symmetric in their variables. Several inequalities involving parametric versions of two Seiffert means, the Neuman-Sándor mean, and the logarithmic means are obtained. It is shown that the last four means belong to the family of the Schwab-Borchardt means. Among inequalities established in this paper some provide generalizations of known results obtained recently by several researchers.

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Paper JMI-07-35

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