

INEQUALITIES AND ASYMPTOTIC EXPANSIONS OF THE WALLIS SEQUENCE AND THE SUM OF THE WALLIS RATIO

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Abstract. The asymptotic expansions for the Wallis sequence is studied in details, and explicit formulae for coefficients are given in their simplest form. This enables improvement of some basic inequalities connected with this sequence. Furthermore we establish identity, inequality and asymptotic expansion for the sum of the Wallis ratio.

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