

## A CERTAIN FUNCTIONAL INEQUALITY DERIVED FROM AN OPERATOR INEQUALITY

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*Abstract.* We will show a certain functional inequality involving fractional powers, making use of the grand Furuta inequality and Tanahashi's argument.

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