

ON k -QUASI-PARANORMAL OPERATORS

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Abstract. For a positive integer k , an operator $T \in B(\mathcal{H})$ is called k -quasi-paranormal if $\|T^{k+1}x\|^2 \leq \|T^{k+2}x\|\|T^kx\|$ for all $x \in \mathcal{H}$, which is a common generalization of paranormal and quasi-paranormal. In this paper, firstly we prove some inequalities of this class of operators; secondly we give a necessary and sufficient condition for T to be k -quasi-paranormal. Using these results, we prove that: (1) if $\|T^{n+1}\| = \|T\|^{n+1}$ for some positive integer $n \geq k$, then a k -quasi-paranormal operator T is normaloid; (2) if E is the Riesz idempotent for an isolated point λ_0 of the spectrum of a k -quasi-paranormal operator T , then (i) if $\lambda_0 \neq 0$, then $E\mathcal{H} = \ker(T - \lambda_0)$; (ii) if $\lambda_0 = 0$, then $E\mathcal{H} = \ker(T^{k+1})$.

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REFERENCES

- [1] M. CHŌ AND K. TANAHASHI, *Isolated point of spectrum of p -hyponormal, log-hyponormal operators*, Integral Equation Operator Theory **43** (2002), 379–384.
- [2] M. CHŌ AND T. YAMAZAKI, *An operator transform from class A to the class of hyponormal operators and its application*, Integral Equations and Operator Theory, **53** (2005), 497–508.
- [3] M. CHŌ AND T. YAMAZAKI, *Erratum to “An operator transform from class A to the class of hyponormal operators and its application”* [Integral Equations and Operator Theory, **53** (2005), 497–508], to appear.
- [4] T. FURUTA, M. ITO AND T. YAMAZAKI, *A subclass of paranormal operators including class of log-hyponormal and several classes*, Sci. Math. **1** (1998), no. 3, 389–403.
- [5] F. GAO AND X. FANG, *On k -quasi-class A operators*, J. Inequal. Appl., **2009** (2009), Article ID 921634, 1–10.
- [6] Y. M. HAN AND W. H. NA, *A note on quasi-paranormal operators*, Mediterr. J. Math. **10** (2013), 383–393.
- [7] J. K. HAN, H. Y. LEE AND W. Y. LEE, *Invertible completions of 2×2 upper triangular operator matrices*, Proc. Amer. Math. Soc. **128** (2000), no. 1, 119–123.
- [8] I. H. JEON AND I. H. KIM, *On operators satisfying $T^*|T^2|T \geq T^*|T|T^2$* , Linear Algebra Appl. **418** (2006), 854–862.
- [9] S. MECHERI, *Bishop’s property (β) and Riesz idempotent for k -quasi-paranormal operators*, Banach J. Math. Anal. **6** (1), (2012), 147–154.
- [10] J. G. STAMPFLI, *Hyponormal operators and spectrum density*, Trans. Amer. Math. Soc. **117** (1965), 469–476.
- [11] K. TANAHASHI AND A. UCHIYAMA, *Isolated point of spectrum of p -quasihyponormal operators*, Linear Algebra Appl. **341** (2002), 345–350.
- [12] K. TANAHASHI, A. UCHIYAMA AND M. CHŌ, *Isolated points of spectrum of (p, k) -quasihyponormal operators*, Linear Algebra Appl. **382** (2004), 221–229.
- [13] K. TANAHASHI, I. H. JEON, I. H. KIM AND A. UCHIYAMA, *Quasinilpotent part of class A or (p, k) -quasihyponormal operators*, Operator Theory: Advances and Applications (birkhäuser) **187** (2008), 199–210.
- [14] A. UCHIYAMA AND K. TANAHASHI, *On the Riesz idempotent of class A operators*, Math. Inequal. Appl. **5**, (2002), no. 2, 291–298.

- [15] A. UCHIYAMA, *On the isolated points of the spectrum of paranormal operators*, Integral Equation Operator Theory **55**, (2006), 145–151.
- [16] A. UCHIYAMA AND K. TANAHASHI, *Bishop's property (β) for paranormal operators*, Oper. Matrices, **4** (2009), 517–524.
- [17] A. UCHIYAMA, *On the isolated points of the spectrum of paranormal operators*, Integral Equation Operator Theory **55**, (2006), 145–151.
- [18] J. YUAN AND G. J., *On (n, k) -quasiparanormal operators*, Studia Math. **209** (3), (2012), 289–301.