

## OPTIMAL BOUNDS FOR NEUMAN-SÁNDOR MEAN IN TERMS OF THE CONVEX COMBINATION OF LOGARITHMIC AND QUADRATIC OR CONTRA-HARMONIC MEANS

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*Abstract.* In this article, we present the least values  $\alpha_1$ ,  $\alpha_2$ , and the greatest values  $\beta_1$ ,  $\beta_2$  such that the double inequalities

$$\alpha_1 L(a, b) + (1 - \alpha_1) Q(a, b) < M(a, b) < \beta_1 L(a, b) + (1 - \beta_1) Q(a, b)$$

$$\alpha_2 L(a, b) + (1 - \alpha_2) C(a, b) < M(a, b) < \beta_2 L(a, b) + (1 - \beta_2) C(a, b)$$

hold for all  $a, b > 0$  with  $a \neq b$ , where  $L(a, b)$ ,  $M(a, b)$ ,  $Q(a, b)$  and  $C(a, b)$  are respectively the logarithmic, Neuman-Sándor, quadratic and contra-harmonic means of  $a$  and  $b$ .

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