

## SCHUR-CONVEXITY, SCHUR GEOMETRIC AND SCHUR HARMONIC CONVEXITIES OF DUAL FORM OF A CLASS SYMMETRIC FUNCTIONS

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**Abstract.** By the properties of Schur-convex function, Schur geometrically convex function and Schur harmonically convex function, Schur-convexity, Schur geometric and Schur harmonic convexities of the dual form for a class of symmetric functions are simply proved. As an application, several inequalities are obtained, some of which extend the known ones.

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