

ON SOME MEANS DERIVED FROM THE SCHWAB–BORCHARDT MEAN II

EDWARD NEUMAN

Abstract. Sharp bounds for four bivariate means derived from the Schwab-Borchardt mean are obtained. The bounding quantities are either geometric or arithmetic convex combinations of two generating means. The four means discussed in this paper have been introduced and studied in [13].

Mathematics subject classification (2010): 26E60, 26D05, 26D07.

Keywords and phrases: Bivariate means, Schwab-Borchardt mean, convex combinations, inequalities.

REFERENCES

- [1] M. ABRAMOWIZ, A. S. STEGUN, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York, 1972.
- [2] H. ALZER, S.-L. QIU, *Monotonicity theorems and inequalities for complete elliptic integrals*, J. Comput. Appl. Math. **172** (2004), 289–312.
- [3] J. M. BORWEIN, P. B. BORWEIN, *Pi and the AGM: A Study in Analytic Number Theory and Computational Complexity*, John Wiley and Sons, New York, 1987.
- [4] B. C. CARLSON, *Algorithms involving arithmetic and geometric means*, Amer. Math. Monthly **78** (1971), 496–505.
- [5] Y.-M. CHU AND B.-Y. LONG, *Bounds of the Neuman-Sándor mean using power and identric means*, Abstr. Appl. Anal., 2013, Article ID 832591, 6 pages.
- [6] Y.-M. CHU, B.-Y. LONG, W.-M. GONG AND Y.-Q. SONG, *Sharp bounds for Seiffert and Neuman-Sándor means in terms of generalized logarithmic means*, J. Inequal. Appl. **2013**, 10 (2013), 13 pages.
- [7] Z.-Y. HE, W.-M. QIAN, Y.-L. JIANG, Y.-Q. SONG AND Y.-M. CHU, *Bounds for the combination of Neuman-Sándor, arithmetic, and second Seiffert means in terms of contra-harmonic mean*, Abstr. Appl. Anal., 2013, Article ID 903982, 5 pages.
- [8] E. NEUMAN, *Inequalities for the Schwab-Borchardt mean and their applications*, J. Math. Inequal. **5**, 4 (2011), 601–609.
- [9] E. NEUMAN, *A note on a certain bivariate mean*, J. Math. Inequal. **6**, 4 (2012), 637–643.
- [10] E. NEUMAN, *A one-parameter family of bivariate means*, J. Math. Inequal. **7**, 3 (2013), 399–412.
- [11] E. NEUMAN, *Sharp inequalities involving Neuman-Sándor and logarithmic means*, J. Math. Inequal. **7**, 3 (2013), 413–419.
- [12] E. NEUMAN, *Inequalities involving certain bivariate means II*, J. Inequal. Spec. Funct., in press.
- [13] E. NEUMAN, *On some means derived from the Schwab-Borchardt mean*, J. Math. Inequal., to appear.
- [14] E. NEUMAN, J. SÁNDOR, *On the Schwab-Borchardt mean*, Math. Pannon. **14**, 2 (2003), 253–266.
- [15] E. NEUMAN, J. SÁNDOR, *On the Schwab-Borchardt mean II*, Math. Pannon. **17**, 1 (2006), 49–59.
- [16] S. PONNUSAMY, M. VUORINEN, *Asymptotic expansions and inequalities for hypergeometric functions*, Mathematika **44** (1997), 43–64.
- [17] W.-M. QIAN AND Y.-M. CHU, *On certain inequalities for Neuman-Sándor mean*, Abstr. Appl. Anal., 2013, Article ID 790783, 6 pages.
- [18] Y.-Q. SONG, W.-M. QIAN, Y.-L. JIANG AND Y.-M. CHU, *Optimal lower generalized logarithmic mean bound for the Seiffert mean*, J. Appl. Math., 2013, Article ID 273653, 5 pages.
- [19] M.-K. WANG, Y.-M. CHU, B.-Y. LIU, *Sharp inequalities for the Neuman-Sándor mean in terms of arithmetic and contra-harmonic means*, preprint, arXiv: 1209.5825 vol 1 [math CA] 26 Sept. 2012.

- [20] M.-K. WANG, Y.-F. QIU AND Y.-M. CHU, *Sharp bounds for Seiffert means in terms of Lehmer means*, J. Math. Inequal. **4** (2010), 581–586.
- [21] A. WITKOWSKI, *Interpolations of Schwab-Borchardt means*, Math. Inequal. Appl. **16**, 1 (2013), 193–206.
- [22] T.-H. ZHAO, Y.-M. CHU, Y.-L. JIANG AND Y.-M. LI, *Best possible bounds for Neuman-Sándor mean by identric, quadratic and contraharmonic means*, Abstr. Appl. Anal., 2013, Article ID 348326, 12 pages.
- [23] T.-H. ZHAO, Y.-M. CHU, B.-Y. LIU, *Optimal bounds for the Neuman-Sándor mean in terms of arithmetic and contra-harmonic means*, Abstract Appl. Anal. Volume **2012**, Article ID 302635, 9 pages.