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PARABOLIC FRACTIONAL MAXIMAL AND INTEGRAL OPERATORS WITH ROUGH KERNELS IN PARABOLIC GENERALIZED MORREY SPACES

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Abstract. Let P be a real $n\times n$ matrix, whose all the eigenvalues have positive real part, $A_t = t^P$, $t>0$, $\gamma = \operatorname{tr} P$ is the homogeneous dimension on $\mathbb{R}^n$ and Ω is an A_t-homogeneous of degree zero function, integrable to a power $s>1$ on the unit sphere generated by the corresponding parabolic metric. We study the parabolic fractional maximal and integral operators $M^P_{\Omega,\alpha}$ and $I^P_{\Omega,\alpha}$, $0<\alpha<\gamma$ with rough kernels in the parabolic generalized Morrey space $\mathscr{M}_{p,\varphi,P}(\mathbb{R}^n)$. We find conditions on the pair (φ_1,φ_2) for the boundedness of the operators $M^P_{\Omega,\alpha}$ and $I^P_{\Omega,\alpha}$ from the space $\mathscr{M}_{p,\varphi_1,P}(\mathbb{R}^n)$ to another one $\mathscr{M}_{q,\varphi_2,P}(\mathbb{R}^n)$, $1<p<q<\infty$, $1/p-1/q=\alpha/\gamma$, and from the space $\mathscr{M}_{1,\varphi_1,P}(\mathbb{R}^n)$ to the weak space $W\mathscr{M}_{q,\varphi_2,P}(\mathbb{R}^n)$, $1\leqslant q<\infty$, $1-1/q=\alpha/\gamma$. We also find conditions on φ for the validity of the Adams type theorems $M^P_{\Omega,\alpha}$, $I^P_{\Omega,\alpha}$: $\mathscr{M}_{p,\varphi^{\frac{1}{p}},P}(\mathbb{R}^n) \to \mathscr{M}_{q,\varphi^{\frac{1}{q}},P}(\mathbb{R}^n), 1<p<q<\infty$.

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