

SOME NOTES ON GREEN–OSHER’S INEQUALITY

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Abstract. In this note we will first focus our attention on the equality case of Green-Osher’s inequality and show that its equality holds if and only if the curve γ is a circle under the assumption that the function $F(x)$ is strictly convex on $(0, +\infty)$. As applications of Green-Osher’s inequality we will then give some new geometric inequalities about convex plane curves. Finally, we will derive an upper bound estimate of Green-Osher’s difference via the method of deforming convex curves into finite circles.

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