

TWO KINDS OF COMPOSITIONS OF HILBERT–HARDY–TYPE INTEGRAL OPERATORS AND THE RELATED INEQUALITIES

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Abstract. By the use of the way of Real and Functional Analysis and estimating the weight functions, we build some lemmas and deduce some Hilbert-Hardy-type integral inequalities. The equivalent forms and the reverses are all considered. Two kinds of Hilbert-Hardy-type integral operators are defined and the composition formulas of the operators are given.

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