

## OPTIMAL ONE-PARAMETER MEAN BOUNDS FOR THE CONVEX COMBINATION OF ARITHMETIC AND LOGARITHMIC MEANS

ZAI-YIN HE, MIAO-KUN WANG AND YU-MING CHU

*Abstract.* We find the greatest value  $p_1 = p_1(\alpha)$  and the least value  $p_2 = p_2(\alpha)$  such that the double inequality  $J_{p_1}(a, b) < \alpha A(a, b) + (1 - \alpha)L(a, b) < J_{p_2}(a, b)$  holds for any  $\alpha \in (0, 1)$  and all  $a, b > 0$  with  $a \neq b$ . Here,  $A(a, b)$ ,  $L(a, b)$  and  $J_p(a, b)$  denote the arithmetic, logarithmic and  $p$ -th one-parameter means of two positive numbers  $a$  and  $b$ , respectively.

*Mathematics subject classification (2010):* 26E60.

*Keywords and phrases:* One-parameter mean, arithmetic mean, logarithmic mean.

### REFERENCES

- [1] M.-Y. SHI, Y.-M. CHU AND Y.-P. JIANG, *Optimal inequalities among various means of two arguments*, Abstr. Appl. Anal., 2009, Article ID 694394, 10 pages.
- [2] Y.-M. CHU AND W.-F. XIA, *Two optimal double inequalities between power mean and logarithmic mean*, Comput. Math. Appl., 2010, **60** (2): 83–89.
- [3] Y.-M. CHU, Y.-F. QIU, M.-K. WANG AND G.-D. WANG, *The optimal convex combination bounds of arithmetic and harmonic means for the Seiffert's mean*, J. Inequal. Appl., 2010, Article ID 436457, 7 pages.
- [4] H. ALZER AND S.-L. QIU, *Inequalities for means in two variables*, Arch. Math., 2003, **80** (2): 201–215.
- [5] H.-Y. GAO AND W.-J. NIU, *Sharp inequalities related to one-parameter and Gini mean*, J. Math. Inequal., 2012, **6** (4): 545–555.
- [6] H.-N. HU, G.-Y. TU AND Y.-M. CHU, *Optimal bounds for Seiffert mean in terms of one-parameter means*, J. Appl. Math., 2012, Article ID 917120, 7 pages.
- [7] P. KAHLIG AND J. MATKOWSKI, *Functional equations involving the logarithmic mean*, Z. Angew. Math. Mech., 1996, **76** (7): 385–390.
- [8] A. O. PITTENGER, *The logarithmic mean in  $n$  variables*, Amer. Math. Monthly, 1985, **92** (2): 99–104.
- [9] G. PÓLYA AND G. SZEGÖ, *Isoperimetric Inequalities in Mathematical Physics*, Princeton University Press, Princeton, 1951.
- [10] P. S. BULLEN, D. S. MITRINOVIĆ AND P. M. VASIĆ, *Means and Their Inequalities*, D. Reidel Publishing Co., Dordrecht, 1988.
- [11] W.-F. XIA, Y.-M. CHU AND G.-D. WANG, *The optimal upper and lower power mean bounds for a convex combination of the arithmetic and logarithmic means*, Abstr. Appl. Anal., 2010, Article ID 604804, 9 pages.