

UNIValENCY OF A NONLINEAR INTEGRAL OPERATOR OF ANALYTIC FUNCTIONS

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Abstract. In this paper, we obtain new univalence conditions for the nonlinear integral operator

$$F_{\alpha}(p)(z) = \left[\alpha \int_0^z u^{\alpha-1} \exp \left(\int_0^u \frac{\beta(p(t)-1)}{t} dt \right) du \right]^{\frac{1}{\alpha}}$$

where $p(z)$ is analytic function in the open unit disk and satisfies $p(0) = 1$, $\alpha \in \mathbb{C}$ with $\operatorname{Re}(\alpha) > 0$ and $\beta \in \mathbb{C}^* = \mathbb{C} \setminus \{0\}$. The numbers of known or new univalence conditions are shown to follow upon specializing the parameters involved in our main results.

Mathematics subject classification (2010): 30C45.

Keywords and phrases: Analytic and univalent functions, integral operator.

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