

A REFINEMENT OF THE JESSEN–MERCER INEQUALITY AND A GENERALIZATION ON CONVEX HULLS IN \mathbb{R}^k

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Abstract. A refinement of the Jessen–Mercer inequality is obtained and shown to be an improvement of the upper bound for the Jessen’s difference given in [12]. Also a generalization of the Jessen–Mercer inequality for convex functions on convex hulls in \mathbb{R}^k is given and demonstrated to be an improvement of the inequalities obtained in [3]. An elegant method of producing n -exponentially convex and exponentially convex functions is applied using the Jessen–Mercer differences. Lagrange and Cauchy mean value type theorems are proved and shown to be useful in studying Stolarsky type means defined by using the Jessen–Mercer differences.

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