

## STRONGLY $\lambda$ -CONVEX FUNCTIONS AND SOME CHARACTERIZATION OF INNER PRODUCT SPACES

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*Abstract.* In this paper we show that each strongly  $\lambda$ -convex function  $f : D \rightarrow \mathbb{R}$  with modulus  $c > 0$ , where  $D$  is a nonempty convex subset of inner product space  $X$  with norm  $\|\cdot\|$ , must be of the form  $g + \|\cdot\|^2$ , where  $g$  is an  $\lambda$ -convex function. Moreover, involving the notion of strongly  $\lambda$ -convexity we get a new characterization of inner product space.

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