

ON MONOTONE ĆIRIĆ QUASI-CONTRACTION MAPPINGS

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Abstract. We prove the existence of fixed points of monotone quasi-contraction mappings in metric and modular metric spaces. This is the extension of Ran and Reurings fixed point theorem for monotone contraction mappings in partially ordered metric spaces to the case of quasi-contraction mappings introduced by Ćirić. The proofs are based on Lemmas ?? and ??, which contain two crucial inequalities essential to obtain the main results.

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