

## ON THE ICOSAHEDRON INEQUALITY OF LÁSZLÓ FEJES-TÓTH

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*Abstract.* In this paper we deal with the problem of finding the maximal volume polyhedra with a prescribed property and inscribed in the unit sphere. We generalize an inequality (called *icosahedron inequality*) of L. Fejes-Tóth which has the following interesting consequence: the regular icosahedron has maximal volume in the class of the polyhedra having twelve vertices and inscribed in the unit sphere. We give an upper bound for the volume of such star-shaped (with respect to the origin) simplicial polyhedra, whose number of faces, and also the list of the maximal edge lengths of the faces are given. As a consequence of this inequality we prove a conjecture which states that the maximal volume polyhedron spanned by the vertices of two regular simplices with common centroid is the cube.

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