

JORDAN-VON NEUMANN TYPE CONSTANT AND FIXED POINTS FOR MULTIVALUED NONEXPANSIVE MAPPINGS

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Abstract. We give some sufficient conditions for the Domínguez-Lorenzo condition in terms of the Jordan-von Neumann type constant, and the coefficient of weak orthogonality. As a consequence, we obtain some sufficient conditions for normal structure and fixed point theorems for multivalued nonexpansive mappings. These fixed point theorems improve some previous results in the recently papers.

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