

## STARLIKENESS OF A CROSS-PRODUCT OF BESSEL FUNCTIONS

HUDA A. AL-KHARSANI, ÁRPÁD BARICZ AND TIBOR K. POGÁNY

**Abstract.** In this paper a necessary and sufficient condition is deduced for the close-to-convexity of a cross-product of Bessel and modified Bessel functions of the first kind and their derivatives by using a result of Shah and Trimble about transcendental entire functions with univalent derivatives, the newly discovered power series and infinite product representation of this cross-product, as well as a slightly modified version of a result of Lorch on the monotonicity of the zeros of the cross-product with respect to the order.

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