

## A METHOD FOR PROVING SOME INEQUALITIES ON MIXED TRIGONOMETRIC POLYNOMIAL FUNCTIONS

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**Abstract.** In this article we present a method for proving a class of inequalities of the form (??). The method is based on the precise approximations of the sine and cosine functions by Maclaurin polynomials of given order. By using this method we present new proofs of some inequalities of C.-P. Chen, W.-S. Cheung [J. Inequal. Appl. 2012:72 (2012)] and Z.-J. Sun, L. Zhu [ISRN Math. Anal. (2011)].

**Mathematics subject classification (2010):** 26D05, 41A10.

**Keywords and phrases:** Trigonometric inequalities, approximations of the sine and cosine, Maclaurin polynomials.

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