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A NOTE ON A WIELANDT TYPE NORM INEQUALITY

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Abstract. As a continuation of recent study on a Wielandt type norm inequality due to Lin [13, Conjecture 3.4], we prove the following result: Let $A \in M_n(\mathbb{C})$ satisfying $0 < m \leqslant A \leqslant M$, and let X and Y be $n \times k$ matrices such that $X^*X = Y^*Y = I_k$ and $X^*Y = 0$. Then for every 2-positive unital linear map Φ, we have

$$\|(\Phi(X^*AY)\Phi(Y^*AY)^{-1}\Phi(Y^*AX))^{\frac{p}{2}}\Phi(X^*AX)^{-\frac{p}{2}}\| \leqslant \begin{cases} \left(\frac{M-m}{M+m}\right)^p \frac{(M^{\frac{p}{2}}+m^{\frac{p}{2}})^2}{4M^{\frac{p}{2}}m^{\frac{p}{2}}} & 1 < p < 2 \\ \frac{(M-m)^p}{4M^{\frac{p}{2}}m^{\frac{p}{2}}} & p \geqslant 2. \end{cases}$$

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