

AN INTRINSIC SQUARE FUNCTION ON WEIGHTED HERZ SPACES WITH VARIABLE EXPONENT

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Abstract. We define new generalized Herz spaces having weight and variable exponent, that is, weighted Herz spaces with variable exponent. We prove the boundedness of an intrinsic square function on those spaces under proper assumptions on each exponent and weight.

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