

IMPROVEMENTS OF BOUNDS FOR THE q -GAMMA AND THE q -POLYGAMMA FUNCTIONS

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Abstract. In this paper, the complete monotonicity property of functions involving the q -gamma function is proven and used to establish sharp inequalities for the q -gamma and the q -polygamma functions for all $q > 0$. These bounds for the q -gamma and the q -polygamma functions refine those given by Salem [17].

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