

## SCHUR CONVEX FUNCTIONS AND THE BONNESEN STYLE ISOPERIMETRIC INEQUALITIES FOR PLANAR CONVEX POLYGONS

JIBING QI AND WEN WANG

*Abstract.* In this note, we continue to investigate Bonnesen-type isoperimetric inequalities for planar convex polygons. We shall first establish some analytic isoperimetric inequalities for a special class of Schur convex functions. Subsequently, by using these analytic isoperimetric inequalities, Bonnesen-type isoperimetric inequalities and related inverse inequalities for the planar convex polygons are obtained.

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