

A GENERALIZED NONLINEAR SUMS–DIFFERENCE INEQUALITY AND ITS APPLICATIONS

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Abstract. In this paper, we established a generalized sums difference inequality with two variables, which included five sums. By using a lemma, we turned the inequality into a common form. We applied our result to boundary value problem of a partial difference equation for boundedness, uniqueness.

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