

ON APPROXIMATION PROPERTIES OF SOME CLASS POSITIVE LINEAR OPERATORS IN q -ANALYSIS

ERSIN ŞİMŞEK AND TUNCAY TUNÇ

Abstract. This paper is concerned with some sequences of the positive linear operators based on q -Calculus. The approximation properties and the rate of convergence of these sequences of q -discrete type is established by means of the modulus of continuity. Moreover we give Voronovskaya-type theorems. Finally we present some applications such as q -Bernstein operators and q -Meyer-König and Zeller operators.

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