

ON THE MATRIX HARMONIC MEAN

MOHAMMAD SABABHEH

Abstract. The main goal of this article is to present new types of inequalities refining and reversing inequalities of the harmonic mean of scalars and matrices. Furthermore, implementing the spectral decomposition of positive matrices, we present a new type of inequalities treating certain harmonic matrix perturbation.

Mathematics subject classification (2010): 15A39, 15B48, 47A30, 47A63.

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