

APPROXIMATION OF FUNCTIONS BY GENUINE BERNSTEIN–DURRMEYER TYPE OPERATORS

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Abstract. Very recently, in [4] Chen et. al introduced and considered a new generalization of Bernstein polynomials depending on a parameter α . As classical Bernstein operators, these operators also provide interpolation at the end points of $[0, 1]$ and they have the linear precision property which means those reproduce the linear functions. In this paper we introduce genuine α -Bernstein-Durrmeyer operators. Some approximation results, which include local approximation, error estimation in terms of Ditzian-Totik modulus of smoothness are obtained. Also, the convergence of these operators to certain functions is shown by illustrative graphics using MAPLE algorithms.

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