

## COMMUTATORS OF MARCINKIEWICZ INTEGRALS WITH ROUGH KERNELS ON HERZ-TYPE HARDY SPACES WITH VARIABLE EXPONENT

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*Abstract.* Let  $\Omega \in L^s(S^{n-1})$  for  $s \geq 1$  be homogeneous functions of degree zero and  $b$  be BMO functions. In this paper, we obtain some boundedness of the Marcinkiewicz integral operator  $\mu_\Omega$  and its commutator  $[b, \mu_\Omega]$  on Herz-type Hardy spaces with variable exponent.

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