

## AN INEQUALITY FOR DISTANCES AMONG FIVE POINTS AND DISTANCE PRESERVING MAPPINGS

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*Abstract.* Using properties of norm and inner product, we prove a new inequality for distances between five points arbitrarily given in an inner product space. Moreover, we investigate the Aleksandrov-Rassias problem by proving that if the distance 1 is contractive and the golden ratio is extensive by a mapping  $f$ , then  $f$  is a linear isometry up to translation.

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