

ON CERTAIN CONJECTURES FOR THE TWO SEIFFERT MEANS

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Abstract. In 2015 Vukšić, by using the asymptotic expansion method, conjectured certain inequalities related to the first and second Seiffert means. In this paper, we prove certain conjectures given by Vukšić.

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