

PROPERTIES OF FUNCTIONS RELATED TO HADAMARD TYPE INEQUALITY AND APPLICATIONS

WEN WANG AND XINQUAN ZHANG

Abstract. The aim of this paper is first to generalize Hadamard's inequality. Further, Schur m -power convex of the associated continuous function of two variables by utilize the Hadamard type inequalities are obtained. As applications, a inequality related special mean is established. And we also improve Jordan's inequality.

Mathematics subject classification (2010): 26D15, 26D10.

Keywords and phrases: Hadamard's inequality, Jordan's inequality, Schur m -power convexity, Schur convexity, mean.

REFERENCES

- [1] Y. CHU, G. WANG AND X. ZHANG, *Schur convexity and hadamard's inequality*, Math. Ineq. Appl., Preprint.
- [2] Y. M. CHU, T. C. SUN, *The Schur harmonic convexity for a class of symmetric functions*, Acta mathematica Scientia, (2010), **30B**(5):1501–1506.
- [3] V. ČULJAK, *A remark on Schur-convexity of the mean of a convex function*, J. Math. Inequal., **9**(4)(2015), 1133–1142.
- [4] S. S. DRAGOMIR, C. E. M. PEARCE, *Selected Topics on Hermite-Hadamard Inequalities and Applications*, RGMIA Monographs, Victoria University, 2000.
- [5] S. S. DRAGOMIR, *Two functions in connection to Hadamard's inequalities*, J. Math. Anal. Appl., **167** (1992), 49–56.
- [6] S. S. DRAGOMIR, *Further properties of some mappings associated with Hermite-Hadamard's inequalities*, Tamkang. J. Math., **34**(1)(2003), 45–57.
- [7] N. ELEZOVIĆ AND J. PEČARIĆ, *A note on Schur-convex fuctions*, Rocky Mountain J. Math. **2000**, **30** (3): 853–856.
- [8] K. Z. GUAN, R. UKE GUAN, *Some properties of a generalized Hamy symmetric function and its applications*, J.Math. Analysis Appl., **376** (2011), 494505.
- [9] R. KLÉN, M. VISURI AND M. VUORINEN, *On Jordan type inequalities for hyperbolic functions*, J. Inequal. Appl. **2010** (2010), Art. ID 362548, 14 pages, doi:10.1155/2010/362548.
- [10] R. B. MANFRINO, R. V. DELGADO, J. A. G. ORTEGA, *Inequalities a Mathematical Olympiad Approach*, Birkhäuser, 2009.
- [11] A. W. MARSHALL, I. OLKIN, B. C. ARNOLD, *Inequalities: theory of majorization and its application (Second Edition)*, New York: Springer Press, 2011.
- [12] M. MERKLE, *Conditions for convexity of a derivative and some applications to the Gamma function*, Aequ. Math., **55** (1998), 273–280.
- [13] M. MERKLE, *Representation of the error terms in Jensen's and some related inequalities with applications*, J. Math. Analysis Appl., **231** (1999), 76–90.
- [14] D. S. MITRINOVIĆ, J. E. PEČARIĆ, A. M. FINK, *Classical and New Inequalities in Analysis*, Kluwer Academic Publishers, Dordrecht.
- [15] C. P. NICULESCU, *The Hermite-Hadamard inequality for log-convex functions*, Nonlinear Analysis, **75** (2012), 662–669.
- [16] M. E. ÖZDEMİR, *A theorem on mappings with bounded derivatives with applications to quadrature rules and means*, Appl. Math. Lett. **13** (2000), 19–25.

- [17] J. E. PEČARIĆ, F. PROSCHAN, Y. L. TONG, *Convex Functions, Partial Orderings and Statistical Applications*, Academic Press, 1991.
- [18] F. QI, D. W. NIU AND B. N. GUO, *Refinements, generalizations, and applications of Jordan's inequality and related problems*, *J. Inequal. Appl.*, **2009** (2009), Art. ID 271923, 52 pages.
- [19] H. N. SHI, *Schur-convex functions related to Hadamard-type inequalities*, *J. Math. Inequal.*, **1**, 1, (2007): 127–136.
- [20] W. T. SULAIMAN, *Schur Convexity and Hadamard's Inequality*, *Global Journal of Sci. frontier Research*, (2011), 11, 1.
- [21] K. L. TSENG, SHIOW-RU HWANG, S. S. DRAGOMIRC, *New Hermite-Hadamard-type inequalities for convex functions (I)*, *Appl. Math. Lett.*, **25** (2012), 1005–1009.
- [22] W. WANG, S. G. YANG, *Schur m-power convexity of generalized Hamy symmetric function*, *J. Math. Inequal.*, 2014, 8(3): 661–667.
- [23] W. WANG, S. G. YANG, *Schur m-power convexity of a class of multiplicatively convex functions and applications*, *Abstract and Applied Analysis*, 2014, Article ID 258108, 12 pages, <http://dx.doi.org/10.1155/2014/258108>
- [24] SH.-H. WU AND L. DEBNATH, *A new generalized and sharp version of Jordan's inequality and its applications to the improvement of the Yang Le inequality*, *Appl. Math. Letters*, **19** (12) (2006), 1378–1384.
- [25] Y. WU, F. QI, *Schur harmonic convexity for difference of some means*, *Analysis*, (2012), **32**:263–270.
- [26] ZH.-H. YANG, *Schur power convexity of the Stolarsky means*, *Publ. Math. Debrecen*, **80**, 1–2 (2012), 43–66 DOI: 10.5486/PMD.2012.4812.
- [27] ZH.-H. YANG, *Schur power convexity of Gini means*, *Bull. Korean Math. Soc.*, **50**, 2 (2013), 485–498.
- [28] ZH.-H. YANG, *Schur power convexity of the daróczy means*, *Math. Inequal. Appl.*, **16**, 3 (2013), 751–762.
- [29] ZH.-H. YANG, *New sharp Jordan type inequalities and their applications*, <http://arxiv.org/abs/1206.5502v1>
- [30] X. M. ZHANG, *Geometrically-Convex Functions*, Anhui Univ. Press, Hefei, 2004 (in Chinese)