

MARTINGALE INEQUALITIES ON HARDY–LORENTZ–KARAMATA SPACES

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Abstract. We establish several martingale inequalities on Hardy-Lorentz-Karamata spaces via atomic decompositions. We also prove that all martingale Hardy-Lorentz-Karamata spaces are equivalent if the stochastic basis is regular.

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