

## SOME RESULTS RELATED TO BESSEL'S INEQUALITY IN INNER PRODUCT SPACES

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*Abstract.* Some inequalities related to the celebrated Bessel's inequality in inner product spaces are given. They complement the results obtained by Boas-Bellman, Bombieri, Selberg and Heilbronn in the middle of the 20<sup>th</sup> century that have been applied for almost orthogonal series and in Number Theory.

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