

## AN EXTENSION OF THE DAVIS–GUT LAW AND LAI LAW

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**Abstract.** Let  $\{X, X_n, n \geq 1\}$  be a sequence of i.i.d. random variables with  $EX = 0$  and  $EX^2 = 1$  and the partial sums  $S_n = \sum_{k=1}^n X_k$ ,  $n \geq 1$ . Assume that  $f(x)$  and  $g(x)$  are positive functions defined on  $[0, \infty)$ . In this short note, under some suitable conditions, we establish the following results

$$\sum_{n=1}^{\infty} f(n)P\{|S_n| > \beta\sqrt{ng(n)}\} < \infty \quad \text{or} \quad = \infty$$

according as

$$\sum_{n=1}^{\infty} \frac{f(n)}{g(n)} \exp\left\{-\frac{\beta^2}{2}g^2(n)(1 + \alpha(n))\right\} < \infty \quad \text{or} \quad = \infty$$

where  $\alpha(n) = EX^2I\{|X| > \sqrt{ng(n)}\}/EX^2I\{|X| \leq \sqrt{ng(n)}\}$ ,  $\beta > 0$ . The results extend and generalize the known Davis–Gut Law and Lai Law.

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### REFERENCES

- [1] P.Y CHEN, Y.C. QI, *Generalized law of the iterated logarithm and its convergence rate*, Stochastic Analysis and Applications, **25**, 1 (2007), 89–103.
- [2] P.Y CHEN, D.C. WANG, *Convergence rates for probabilities of moderate deviations for moving average processes*, Acta Mathematica Sinica, English Series, **24**, 4 (2008), 611–622.
- [3] J.A. DAVIS, *Convergence rates for the law of the iterated logarithm*, Annals of Mathematical Statistics, **39**, 5 (1968), 1479–1485.
- [4] A. GUT, *Convergence rates for probabilities of moderate deviations for sums of random variables with multidimensional indices*, Annals of Probability, **8**, 2 (1980), 298–313.
- [5] T.L. LAI, *Limit theorems for delayed sums*, Annals of Probability, **2**, 3 (1974), 432–440.
- [6] D.L. LI, *Convergence rates for law of iterated logarithm for B-valued random variables*, Science in China Ser A, **4**, (1991), 395–404.
- [7] D.L. LI, X.C. WANG, M.B. RAO, *Some results on convergence rates for probabilities of moderate deviations for sums of random variables*, International Journal of Mathematics and Mathematical Sciences, **15**, 3 (1992), 131–149.
- [8] D.L. LI, A. ROSALSKY, *A supplement to the Davis–Gut law*, Journal of Mathematical Analysis and Applications, **330**, 2 (2007), 1488–1493.
- [9] X.D. LIU, H.Y. QIAN, L.Q. CAO, *The Davis–Gut law for moving average processes*, Statistics and Probability Letters, **104**, (2015), 1–6.
- [10] X.D. LIU, H. GUO, *A note on the Davis–Gut law*, Statistics and Probability Letters, **105**, 3 (2015), 163–167.
- [11] X.D. LIU, J. MENG, *The Davis–Gut law and Lai law for finitely inhomogeneous walks*, Journal of Mathematical inequalities, **11**, 1 (2017), 281–289.
- [12] S.V. NAGAEV, *Some limit theorems for large deviations*, Theory of Probability and Its Applications, **10**, (1965), 231–254.