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INEQUALITIES ARISING FROM GENERALIZED EULER–TYPE CONSTANTS MOTIVATED BY LIMIT SUMMABILITY OF FUNCTIONS

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Abstract. Limit summability of real functions was introduced by M.H. Hooshmand in 2001. In order to study derivation of the limit summand function, he has introduced a functional sequence corresponding to a given function f with $D_f \supseteq \mathbb{N}^*$ that is related to the Euler-type constants. In the way, we prove two main criteria for its convergence together with an extensive inequality between the limit summand function and the generalized Euler-type constants. The main inequality is also extended whenever f is a convex or concave function. Among other things, we obtain some inequalities for many special functions such as the gamma, digamma and zeta functions.

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