

## ON AN EQUIVALENT PROPERTY OF A REVERSE HILBERT–TYPE INTEGRAL INEQUALITY RELATED TO THE EXTENDED HURWITZ–ZETA FUNCTION

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*Abstract.* We study some equivalent conditions of a reverse Hilbert-type integral inequality with a particular non-homogeneous kernel and a best possible constant factor related to the extended Hurwitz-zeta function. Some equivalent conditions of a reverse Hilbert-type integral inequality with the particular homogeneous kernel are deduced. We also consider some particular cases.

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