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ORLICZ–FRACTIONAL MAXIMAL OPERATORS ON WEIGHTED L^p SPACES

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Abstract. Necessary and sufficient conditions for weight norm inequalities on Lebesgue spaces to hold are given in the scale of Orlicz spaces for the fractional Orlicz maximal operators which generalizes the fractional maximal operators. A similar argument for the Orlicz maximal operators is due to Pérez, who generalizes for the Fefferman–Stein inequality. The main result is the Fefferman–Stein inequality for the fractional maximal operators of the Sawyer type and the Hardy–Littlewood–Sobolev type. In this paper, we establish that the L^p-boundedness and the Fefferman–Stein type inequality of Orlicz maximal operator are essentially equivalent to the Saywer type inequality for the fractional Orlicz maximal operators. These inequalities are stronger than the Hardy–Littlewood–Sobolev type inequalities. More generally, we consider several mixed strong type inequalities for the ordinary and generalized fractional Orlicz maximal operators. As an application, we investigate the weight norm inequalities of the commutator $[b,I_\alpha]$, where $b \in \mathrm{BMO}(\mathbb{R}^n)$, and I_α the fractional integral operator.

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