

## BUZANO'S INEQUALITY IN ALGEBRAIC PROBABILITY SPACES

J. AGREDO, Y. LEON, J. OSORIO AND A. PEÑA

*Abstract.* We obtain a generalization of Buzano's inequality of vectors in Hilbert spaces, using the theory of algebraic probability spaces. In particular, we extend a result of Dragomir given in [?]. Applications for numerical inequalities for  $n$ -tuples of bounded linear operators and functions of operators defined by double power series are also generalized.

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