

ON INNER AND OUTER RADII IN MINKOWSKI SPACES

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Abstract. Some sharp bounds for the inner radius and the outer radius of the unit ball of a Minkowski space with respect to its isoperimetrix are known. To find more such bounds is a challenging problem. Related to this, we derive new relations between inner and outer radii as well as cross-section measures for the Holmes-Thompson and Busemann measures.

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